Response of Moderately Thick Laminated Cross-Ply Composite Shells Subjected to Random Excitation

Isaac Elishakoff*
Technion-Israel Institute of Technology, Haifa, Israel
Gabriel Cederbaum†
Tel Aviv University, Tel Aviv, Israel
and
Liviu Librescu‡
Virginia Polytechnic Institute and State University, Blacksburg, Virginia

This study deals with the dynamic response of transverse shear deformable laminated shells subjected to random excitation. The analysis encompasses the following problems: 1) the dynamic response of circular cylindrical shells of finite length excited by an axisymmetric uniform ring loading, stationary in time, and 2) the response of spherical and cylindrical panels subjected to stationary random loadings with uniform spatial distribution. The associated equations governing the structural theory of shells are derived upon discarding the classical Love-Kirchhoff (L-K) assumptions. In this sense, the theory is formulated in the framework of the first-order transverse shear deformation theory (FSDT).

Introduction

IN 1980, Witt and Sobczyk[1] pioneered the study of random vibrations of laminated plates. In particular, the cross-ply plate in cylindrical bending, subjected to a time-wise stationary load, was considered. Later, Witt[2] generalized this study to the two-dimensional problem. To the best of our knowledge, there are no works on the probabilistic response of the laminated shells. This paper attempts to close this gap.

This study concerns the determination of the contribution played by cross-correlation terms in the evaluation of the mean-square velocity of the shell. As is well known,[3] these terms are generally neglected with respect to the autocorrelation terms. It is shown that, by their incorporation, the maximum mean-square velocity response may exceed its counterpart without the cross-correlation in a dramatic manner. This can be viewed as a generalization of earlier results obtained by Elishakoff et al.,[4] in the case of homogeneous cylinders. In addition, the analysis herein concerns also the determination of the mean-square displacement in the case of cylindrical and spherical panels. For these cases, comparisons of the obtained results with their classical (L-K) counterparts are performed.

Formulation

Following the first-order shear deformation theory (FSDT)[5-7] (see also Ref. 8), the displacement field is given by

$$\begin{aligned} U_1(x,y,z,t) &= U(x,y,t) + z\psi_x(x,y,t) \\ U_2(x,y,z,t) &= V(x,y,t) + z\psi_y(x,y,t) \\ U_3(x,y,t) &= W(x,y,t) \end{aligned} \tag{1}$$

where x and y are the longitudinal and circumferential directions in the case of a complete cylinder, and lines of curvature in the case of panels, z in the radial direction; ψ_x and ψ_y are the rotations of the reference surface, $z = 0$, about the y and x coordinate axes, respectively; and U, V, and W are the displacements of the point $(x,y,0)$.

Substituting Eq. (1) into the strain-displacement relations of an orthogonal curvilinear coordinate system, one obtains

$$\begin{aligned} &\epsilon_1 = \epsilon_1^0 + zK_1, \qquad \epsilon_2 = \epsilon_2^0 + zK_2, \qquad \epsilon_3 = 0 \\ &\epsilon_4 = \epsilon_4^0, \qquad \epsilon_5 = \epsilon_5^0, \qquad \epsilon_6 = \epsilon_6^0 + zK_6 \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\epsilon_1^0 = U_{,x} + (W/R_1), \quad \epsilon_2^0 = V_{,y} + (W/R_2), \quad \epsilon_6^0 = V_{,x} + U_{,y} \\ &\epsilon_4^0 = W_{,y} + \psi_y - (V/R_2), \qquad \epsilon_5^0 = W_{,x} + \psi_x - (U/R_1) \\ &K_1 = \psi_{x,x}, \qquad K_2 = \psi_{y,y} \\ &K_6 = \psi_{y,x} + \psi_{x,y} + \tfrac{1}{2}[(1/R_2) - (1/R_1)](V_{,x} - U_{,y}) \end{aligned}$$

and R_i is the radius to the midsurface of the i direction: in a cylinder shell $R_1 = \infty$, $R_2 = R$, whereas in a spherical shell $R_1 = R_2 = R$.

The stress-strain relations for the ℓth orthotropic lamina in the material coordinate axes are given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & & & \\ \bar{C}_{12} & \bar{C}_{22} & & 0 & \\ & & \bar{C}_{44} & & \\ & 0 & & \bar{C}_{55} & \\ & & & & \bar{C}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{Bmatrix} \tag{3}$$

where $\bar{C}_{ij}$ are the plane-stress reduced material stiffnesses of the lamina

$$\begin{aligned} &\bar{C}_{11} = E_1/\Omega, \qquad \bar{C}_{22} = E_2/\Omega, \qquad \bar{C}_{12} = \nu_{12}E_2/\Omega \\ &\bar{C}_{44} = G_{23}, \qquad C_{55} = G_{13}, \qquad \bar{C}_{66} = G_{12}, \qquad \Omega = 1 - \nu_{12}\nu_{21} \end{aligned}$$

The constitutive relations are

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \epsilon_j^0 \\ K_j \end{Bmatrix} \qquad (i,j = 1,2,6)$$

$$\{Q_i\} = k^2[A_{ij}]\{\epsilon_j^0\} \qquad (i,j = 4,5) \tag{4}$$

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*Professor, Faculty of Aerospace Engineering.
†Research Associate (Ph.D.), Department of Solid Mechanics, Materials and Structures.
‡Professor, Department of Engineering Science and Mechanics.

where (for a cross-ply laminate)

$$(A_{ij},B_{ij},D_{ij}) = \sum_{\ell=1}^{N} \int_{z_{\ell-1}}^{z_\ell} \bar{C}_{ij}\,(1,z,z^2)\,\mathrm{d}z \qquad (i,j = 1,2,6)$$

$$A_{ij} = \sum_{\ell=1}^{N} \int_{z_{\ell-1}}^{z_\ell} \bar{C}_{ij}\,\mathrm{d}z \qquad (i,j = 4,5)$$

and k^2 is the shear correction factor, equal to 5/6.[9] The governing equations are then written, in the absence of body forces, as

$$N_{1,x} + (N_6 + T_0 M_6)_{,y} + \frac{Q_5}{R_1} = \left(I_1 + \frac{2}{R_1} I_2\right)(\ddot{U} + C\dot{U}) + \left(I_2 + \frac{1}{R_2} I_3\right)(\ddot{\psi}_x + C\dot{\psi}_x)$$

$$N_{2,y} + (N_6 - T_0 M_6)_{,x} + \frac{Q_4}{R_2} = \left(I_1 + \frac{2}{R_2} I_2\right)(\ddot{V} + C\dot{V}) + \left(I_2 + \frac{1}{R_2} I_3\right)(\ddot{\psi}\psi_y + C\dot{\psi}_y)$$

$$Q_{5,x} + Q_{4,y} - \left(\frac{N_1}{R_1} + \frac{N_2}{R_2}\right) + q_z = I_1(\ddot{W} + C\dot{W})$$

$$M_{1,x} + M_{6,y} - Q_5 = I_3(\ddot{\psi}_x + C\dot{\psi}_x) + \left(I_2 + \frac{1}{R_1} I_3\right)(\ddot{U} + C\dot{U})$$

$$M_{2,x} + M_{6,x} - Q_4 = I_2(\ddot{\psi}_y + C\dot{\psi}_y) + \left(I_2 + \frac{1}{R_2} I_3\right)(\ddot{V} + C\dot{V}) \tag{5}$$

where

$$T_0 = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$(I_1,I_2,I_3) = \sum_{\ell=1}^{N} \int_{z_{\ell-1}}^{z_\ell} \rho^{(\ell)}(1,z,z^2)\,\mathrm{d}z$$

C is the damping factor, $\rho^{(\ell)}$ the mass density of the ℓth layer, and N the total number of layers.

Case 1: Finite Circular Cylinder Subjected to Uniform Ring Load

Because of symmetry in the circumferential direction, and for symmetric cross-ply laminate, the governing equations (5) become

$$N_{1,x} = I_1(\ddot{U} + C\dot{U})$$
$$Q_{5,x} - (N_2/R) + q_z = I_1(\ddot{W} + C\dot{W})$$
$$M_{1,x} - Q_5 = I_3(\ddot{\psi}_y + C\dot{\psi}_y) \tag{6}$$

The boundary conditions for a close, finite cylinder, simply supported at its edges by shear diaphragms, are

$$N_1 = M_1 = W = 0 \qquad \text{at } x = 0,L \tag{7}$$

and the solution functions and the transverse load functions, in order to obtain exact solution, are given by

$$U(x,t) = \sum_m \hat{U}_m \cos\alpha x\, T_m(t)2 \equiv \sum_m U_m T_m$$
$$W(x,t) = \sum_m \hat{W}_m \sin\alpha x\, T_m(t) \equiv \sum_m W_m T_m$$
$$\psi_x(x,t) = \sum_m \hat{X}_m \cos\alpha x\, T_m(t) \equiv \sum_m X_m T_m \tag{8}$$
$$q_z(x,t) = \sum_m q_m \sin\alpha x\, F_m(t) \tag{9}$$

where $\alpha = m\pi/L$; U_m, W_m, X_m are the coefficients of the natural mode shapes associated with the free vibration problem; $T_m(t)$ denote the generalized coordinates; and q_m are the Fourier coefficients.

For the free vibration problem, $F_m(t) \equiv 0$, $C \equiv 0$, $T_m(t) = e^{i\omega_m t}$ $(i = \sqrt{-1})$, and, using Eq. (8) in Eqs. (6), the eigenvalue problem is obtained as

$$\Big[[K]_m - \omega_m^2 [M]\Big]\{\Delta\}_m = \{0\} \tag{10}$$

where

$$\{\Delta\}_m^T = \{\hat{U}_m, \hat{W}_m, \hat{X}_m\}$$

The elements of the symmetric 3×3 $[K]$ and $[M]$ matrices are

$$\begin{array}{ll} k_{11} = A_{11}\alpha^2 & M_{11} = I_1 \\ k_{12} = -A_{12}\,(\alpha/R) & M_{12} = 0 \\ k_{13} = 0 & M_{13} = 0 \\ k_{22} = A_{22}(1/R^2) + A_{55}\alpha^2 & M_{22} = I_1 \\ k_{23} = A_{55}\alpha & M_{23} = 0 \\ k_{33} = D_{11}\alpha^2 + A_{55} & M_{33} = I_3 \end{array} \tag{11}$$

The orthogonality condition[10] is

$$\left(\omega_m^2 - \omega_n^2\right)\int_0^L \{I_1[U_m U_n + W_m W_n] + I_3 x_m x_n\}\,\mathrm{d}x = 0 \tag{12}$$

and the norm (generalized mass) is defined as

$$J_m = \int_0^L \{I_1[U_m^2 + W_m^2] + I_3 x_m^2\}\,\mathrm{d}x \tag{13}$$

Using the modal analysis technique, the decoupled differential equation for the generalized coordinates $T_m(t)$ is obtained as

$$\ddot{T}_m(t) + C\dot{T}_m(t) + \omega_m^2 T_m(t) = (1/J_m)F_m(t) \tag{14}$$

where

$$C = 2\xi_m\omega_m, \qquad F_m(t) = \int_0^L W_m q_z(x,t)\,\mathrm{d}x$$

For the case of homogeneous initial conditions, the solution of Eq. (14) is

$$T_m(t) = \frac{1}{J_m}\int_0^L F_m(\tau)h_m(t-\tau)\,\mathrm{d}\tau \tag{15}$$

and by Eqs. (8), the transverse displacement is expressed as[3]

$$W(x,t) = \sum_m W_m \frac{1}{J_m}\int_{-\infty}^{t} F_m(\tau)h_m(t-\tau)\,\mathrm{d}\tau$$
$$= \sum_m W_m \frac{1}{J_m}\int_{-\infty}^{\infty} \bar{F}_m(\omega)H_m(\omega)e^{i\omega t} \tag{16}$$

where

$$\bar{F}_m(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_m(t)e^{-i\omega t}\,\mathrm{d}t$$

whereas $H_m(\omega)$ is the complex frequency response function associated with the mth mode:

$$H_m(\omega) = 1/(\omega_m^2 - \omega^2 + 2i\xi_m\omega_m\omega) \equiv 1/L_m(\omega) \tag{17}$$

The correlation function of the transverse displacement is

$$R_w(x_1,x_2,t_1,t_2) = E[W(x_1,t_1)W(x_2,t_2)] \tag{18}$$

and, for the stationary excitation with zero mean,

$$R_w(x_1,x_2,\tau) = \sum_m\sum_n W_m(x_1)W_n(x_2) \times \int_{-\infty}^{\infty} S_{Q_mQ_n}(\omega)\, H_m(\omega)H_n^*(\omega)e^{i\omega\tau}\,\mathrm{d}\omega \tag{19}$$

where

$$S_{Q_mQ_n}(\omega) = \frac{1}{J_mJ_n}\int_0^L\int_0^L S_F(x_1,x_2,\omega)W_m(x_1)W_n(x_2)\,\mathrm{d}x_1\,\mathrm{d}x_2 \tag{20}$$

whereas

$$S_F(x_1,x_2,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_F(x_1,x_2,\tau)e^{-i\omega\tau}\,\mathrm{d}\tau$$

denotes the cross-spectral density function of the applied load.

For the case of a uniform ring load applied at $x = a$, and when its random function represents band-limited white noise with cutoff frequency ω_c,[3]

$$E[F(t)] = 0$$

$$S_F(\omega) = \begin{cases} S_0 & \text{for } |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

The correlation function of the applied load is given by

$$R_F(x_1,x_2,\tau) = R(\tau)\delta(x_1-a)\delta(x_2-a) \tag{22}$$

and its cross-spectral density function

$$S_F(x_1,x_2,\omega) = S_F(\omega)\delta(x_1-a)\delta(x_2-a) \tag{23}$$

so that

$$S_{Q_mQ_n}(\omega) = \frac{1}{J_mJ_n}\int_0^L\int_0^L\left\{S_F(\omega)\delta(x_1-a)\delta(x_2-a) \times \sin\frac{m\pi}{L}x_1\,\sin\frac{n\pi}{L}x_2\right\}\mathrm{d}x_1\,\mathrm{d}x_2 = \frac{S_F(\omega)}{J_mJ_n}\sin\frac{m\pi}{L}a\,\sin\frac{n\pi}{L}a \tag{24}$$

The mean square of the transverse displacement is[3]

$$E[W^2(x,x,\tau)] = \sum_{m,n} S_{Q_mQ_n}(\omega)\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)I_{mn} \tag{25}$$

where

$$I_{mn} = \int_{-\infty}^{\infty} H_m(\omega)H_n^*(\omega)\,\mathrm{d}\omega = \int_{-\omega_c}^{\omega_c}\frac{\mathrm{d}\omega}{(\omega_m^2-\omega^2+2i\xi_m\omega_m\omega)(\omega_n^2-\omega^2-2i\xi_n\omega_n\omega)}$$

$$= \frac{P(\omega_m,\omega_n,\omega_c)+P(\omega_n,\omega_m,\omega_c)}{(\omega_m^2-\omega_n^2)^2+2\beta^2(\omega_m^2+\omega_n^2)}$$

in which $\beta = \xi_m\omega_m$, and

$$P(\omega_m,\omega_n,\omega_c) = \frac{\omega_m^2-\omega_n^2-\beta^2}{2(\omega_m^2-\beta^2/4)}\ln\left[\frac{\omega_c^2+\omega_m^2-2\omega_c(\omega_m^2-\beta^2/4)^{1/2}}{\omega_c^2+\omega_m^2+2\omega_c(\omega_m^2-\beta^2/4)^{1/2}}\right] + 2\beta\left\{\tan^{-1}\left[\frac{\omega_c-(\omega_m^2-\beta^2/4)^{1/2}}{\beta/2}\right] + \tan^{-1}\left[\frac{\omega_c+(\omega_m^2-\beta^2/4)^{1/2}}{\beta/2}\right]\right\}$$

The mean square of the velocity is obtained in the same way so that

$$E[V^2(x,x,\tau)] = \sum_{m,n} S_{Q_mQ_n}(\omega)\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)I'_{mn} \tag{26}$$

where

$$I'_{mn} = \frac{P'(\omega_m,\omega_n,\omega_c)+P'(\omega_n,\omega_m,\omega_c)}{(\omega_m^2-\omega_n^2)^2+2\beta^2(\omega_m^2+\omega_n^2)}$$

and

$$P'(\omega_m,\omega_n,\omega_c) = \frac{\omega_m^2(\omega_m^2-\omega_n^2)+\beta^2(\omega_m^2+\omega_n^2)/2}{2[\omega_m^2-\beta^2/4]^{1/2}} \times \ln\left[\frac{\omega_c^2+\omega_m^2-2\omega_c(\omega_m^2-\beta^2/4)^{1/2}}{\omega_c^2+\omega_m^2+2\omega_c(\omega_m^2-\beta^2/4)^{1/2}}\right] + \beta(\omega_m^2+\omega_n^2)\left\{\tan^{-1}\left[\frac{\omega_c-(\omega_m^2-\beta^2/4)^{1/2}}{\beta/2}\right] + \tan^{-1}\left[\frac{\omega_c+(\omega_m^2-\beta^2/4)^{1/2}}{\beta/2}\right]\right\}$$

Following Ref. 3, Eq. (26) can be approximated as

$$E[V^2(x,x,\tau)] = \sum_{m,n=1}^{N_c} S_{Q_mQ_n}(\omega)\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)I'_{mn} \tag{27}$$

where N_c is the largest values of m, so that $\omega_m \le \omega_c$. This yields

$$E[V^2(x,x,\tau)] \simeq S_1(x,0) + S_2(x,0) \tag{28}$$

where

$$S_1(x,0) = \sum_{m=1}^{N_c}\frac{S_0}{J_m^2}\sin^2\left(\frac{m\pi}{L}a\right)\sin^2\left(\frac{m\pi}{L}x\right)I'_{mn} \tag{29}$$

represents the autocorrelation terms, and

$$S_2(x,0) = \sum_{\substack{m,n=1\\ m\neq n}}^{N_c}\frac{S_0}{J_mJ_n}\sin\left(\frac{m\pi}{L}a\right)\sin\left(\frac{n\pi}{L}a\right) \times \sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)I'_{mn} \tag{30}$$

represents the cross-correlation terms.

Numerical Results and Discussion

Let us define the material properties and dimensions of the cylindrical shells. Consider a four-layer cross-ply laminate [0°,

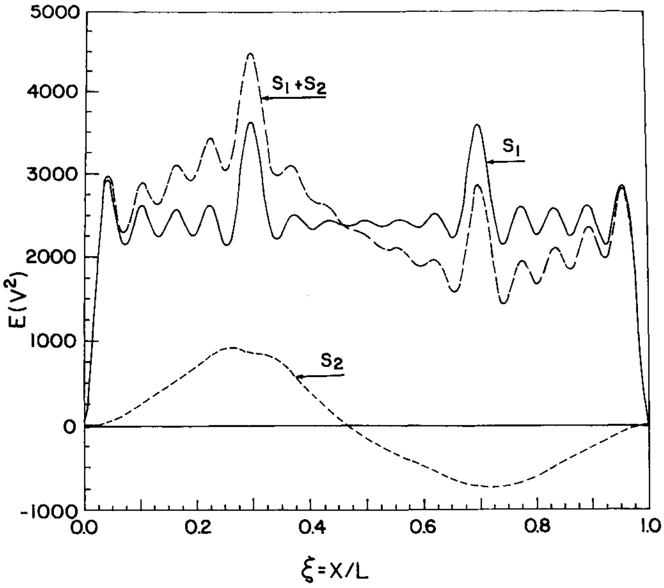

Fig. 1 Mean-square velocity vs $\xi = x/L$, $h/L = 0.01$, $R/L = 0.5$.

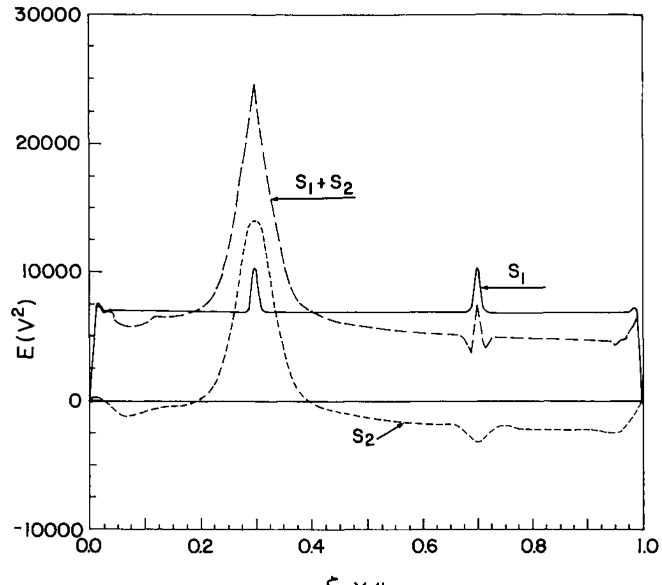

Fig. 2 Mean-square velocity vs $\xi = x/L$, $h/L = 0.001$, $R/L = 0.5$.

90°, 90°, 0°], with the following engineering constants of each ply:

$$E_1 = 19.2 \text{ MSI}, \qquad G_{23} = 0.523 \text{ MSI}$$
$$E_2 = 1.56 \text{ MSI}, \qquad \nu_{12} = 0.24$$
$$G_{12} = 0.82 \text{ MSI}, \qquad \rho = 0.075 \text{ PCI}$$

where 1 MSI = 1 mega lb/in.^{2}; 1 PCI = 1 lb/in.^{3}.

The contribution of the cross-correlation terms to the total mean square of the velocity are demonstrated in Figs. 1–3 for the cases

1) $h/L = 0.01$, $R/L = 0.5$
2) $h/L = 0.001$, $R/L = 0.5$
3) $h/L = 0.0003$, $R/L = 0.5$

The values of S_0 and L are equal to 1. The loading is represented as in Ref. 4, as a ring loading at the cross section $a/L = 0.3$, representing random process, in particular the

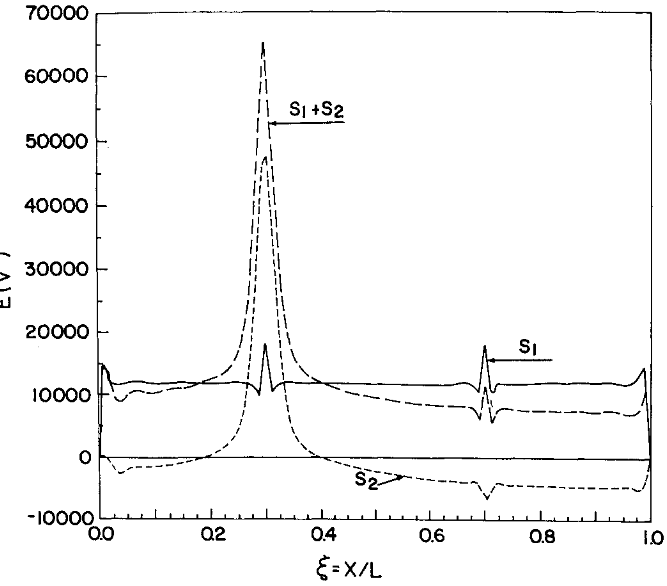

Fig. 3 Mean-square velocity vs $\xi = x/L$, $h/L = 0.0003$, $R/L = 0.5$.

band-limited white noise, where $\omega_c = 7\omega_1$. In each figure, line 1 represents the sum of the autocorrelation terms (s_1), line 2 represents the sum of the cross-correlation terms (S_2), and line 3 is the total sum; N_c was found to be 17 in the first case, 48 in the second, and 87 in the third.

From the figures, it can be seen that the S_1 curve is always symmetric about the midlength of the cylinder ($x = 0.5L$), and non-negative. S_2 has no symmetric properties, but its spatial average is zero. The part of the cross-correlation terms in the total mean square is increased as h/L decreases. The maximum value of the ratio $E[V^2]/S_1$ is 1.233 in the first case, 2.338 in the second, and 3.818 in the third.

Case 2: Cylindrical and Spherical Panels Subjected to Uniformly Distributed Load

The cases of symmetric (0°, 90°, 90°, 0°) and antisymmetric (0°, 90°) cross-ply laminated panels will be investigated herein.

The boundary conditions for these simply supported panels are[6]

$$N_1 = M_1 = W = V = \psi_y = 0 \qquad \text{at } x = 0,a$$
$$N_2 = M_2 = W = U = \psi_x = 0 \qquad \text{at } y = 0,b \tag{31}$$

and the solution function and the transverse load function are given by

$$U(x,y,t) = \sum_{m,n} \hat{U}_{mn} \cos\alpha x \sin\beta y T_{mn}(t) \equiv \sum_{m,n} U_{mn} T_{mn}$$
$$V(x,y,t) = \sum_{m,n} \hat{V}_{mn} \sin\alpha x \cos\beta y T_{mn}(t) \equiv \sum_{m,n} V_{mn} T_{mn}$$
$$W(x,y,t) = \sum_{m,n} \hat{W}_{mn} \sin\alpha x \sin\beta y T_{mn}(t) \equiv \sum_{m,n} W_{mn} T_{mn}$$
$$\psi_x(x,y,t) = \sum_{m,n} \hat{X}_{mn} \cos\alpha x \sin\beta y T_{mn}(t) \equiv \sum_{m,n} X_{mn} T_{mn}$$
$$\psi_y(x,y,t) = \sum_{m,n} \hat{Y}_{mn} \sin\alpha x \cos\beta y T_{mn}(t) \equiv \sum_{m,n} Y_{mn} T_{mn} \tag{32}$$

$$q_z(x,y,t) = \sum_{m,n} q_{mn} \sin\alpha x \sin\beta y \, F_{mn}(t) \tag{33}$$

where

$$\alpha \equiv m\pi/a, \qquad \beta \equiv n\pi/b$$

For cross-ply laminates, the elements of the 5×5 $[K]$ and

$[M]$ symmetric matrices are

$$k_{11} = A_{11}\alpha^2 + A_{66}\beta^2 + D_{66}\beta^2T_0^2 + 2B_{66}\beta^2T_0 + A_{55}\,(1/R_1^2)$$

$$k_{12} = (A_{12} + A_{66} - D_{66}T_0^2)\alpha\beta$$

$$k_{13} = \left(A_{11}\frac{1}{R_1} + A_{12}\frac{1}{R_2} + A_{55}\frac{1}{R_1}\right)\alpha$$

$$k_{14} = B_{11}\alpha^2 + B_{66}\beta^2 + D_{66}\beta^2T_0 - A_{55}\,(1/R_1)$$

$$k_{15} = (B_{12} + B_{66} + D_{66}T_0)\alpha\beta$$

$$k_{22} = A_{66}\alpha^2 + D_{66}\alpha^2T_0^2 + A_{22}\beta^2 - 2B_{66}\alpha^2T_0 + A_{44}(1/R_2^2)$$

$$k_{23} = -\left(A_{12}\frac{1}{R_1} + A_{22}\frac{1}{R_2} + A_{55}\frac{1}{R_1}\right)\beta$$

$$k_{24} = (B_{66} + B_{12} - D_{66}T_0)\alpha\beta$$

$$k_{25} = (B_{66} - D_{66}T_0)\alpha^2 + B_{22}\beta^2 - A_{44}(1/R_2)$$

$$k_{33} = A_{55}\alpha^2 + A_{44}\beta^2 + \left(A_{11}\frac{1}{R_1} + A_{12}\frac{1}{R_2}\right)\frac{1}{R_1} + \left(A_{12}\frac{1}{R_1} + A_{22}\frac{1}{R_2}\right)\frac{1}{R_2}$$

$$k_{34} = -\left(B_{11}\frac{1}{R_1} + B_{12}\frac{1}{R_2} - A_{55}\right)\alpha$$

$$k_{35} = -\left(B_{12}\frac{1}{R_1} + B_{22}\frac{1}{R_2} - A_{44}\right)\beta$$

$$k_{44} = D_{11}\alpha^2 + D_{66}\beta^2 + A_{55}$$

$$k_{45} = (D_{12} + D_{66})\alpha\beta$$

$$k_{55} = D_{66}\alpha^2 + D_{22}\beta^2 + A_{44}$$

$$M_{11} = I_1 + (2/R_1)\,I_2, \qquad M_{14} = I_2 + (1/R_1)I_3$$

$$M_{22} = I_1 + (2/R_2)I_2, \qquad M_{25} = I_2 + (1/R_2)I_3$$

$$M_{33} = I_1$$

$$M_{44} = I_3$$

$$M_{55} = I_3 \tag{34}$$

The norm in this case is

$$J_{mn} = \int_0^a\int_0^b\Bigg\{I_1\left[U_{mn}^2 + V_{mn}^2 + W_{mn}^2\right] + 2I_2\left(U_{mn}X_{mn} + V_{mn}Y_{mn} + \frac{1}{R_1}U_{mn}^2 + \frac{1}{R_2}V_{mn}^2\right) + I_3\left[X_{mn}^2 + Y_{mn}^2 + 2\left(\frac{1}{R_1}U_{mn}X_{mn} + \frac{1}{R_2}V_{mn}Y_{mn}\right)\right]\Bigg\}\,\mathrm{d}x\,\mathrm{d}y \tag{35}$$

(In the case of symmetric laminate, $B_{ij} = I_2 = 0$.)

For the case in which the applied load is uniformly distributed over the panel and in which its correlation function is given by

$$R_F(\tau) = \sigma^2 e^{-\gamma|\tau|} \tag{36}$$

so that[3]

$$S_F(\omega) = \frac{\sigma^2}{\gamma\pi}\frac{\gamma^2}{\gamma^2 + \omega^2} \tag{37}$$

one obtains

$$S_{Q_{mn}Q_{pq}}(\omega) = \frac{S_F(\omega)}{J_{mn}J_{pq}}\int_0^a\int_0^b\int_0^a\int_0^b \times\left\{\sin\frac{m\pi}{a}x_1\sin\frac{n\pi}{b}y_1\sin\frac{p\pi}{a}x_2\sin\frac{q\pi}{b}y_2\right\}\mathrm{d}y_1\,\mathrm{d}x_1\,\mathrm{d}y_2\,\mathrm{d}x_2 = \frac{16}{mnpq}\frac{S_F(\omega)}{J_{mn}J_{pq}} \quad m,n,p,q = 1,3,5,\ldots, \tag{38}$$

The mean square of the transverse displacement, at the point $x = a/2$, $y = b/2$, is

$$E\left[W^2\left(\frac{a}{2},\frac{b}{2},\frac{a}{2},\frac{b}{2},0\right)\right] = \sum_{m,n}\sum_{p,q}\frac{16}{mnpqJ_{mn}J_{pq}}\sin\frac{m\pi}{2} \times\sin\frac{n\pi}{2}\sin\frac{p\pi}{2}\sin\frac{q\pi}{2}\int_{-\infty}^{\infty}\frac{\sigma^2}{\gamma\pi}\frac{\gamma^2}{\gamma^2+\omega^2} \times\frac{\mathrm{d}\omega}{(\omega_{mn}^2 - \omega^2 + 2i\xi_{mn}\omega_{mn}\omega)(\omega_{pq}^2 - \omega^2 - 2i\xi_{pq}\omega_{pq}\omega)} \tag{39}$$

Since the damping is light and, moreover, only the frequencies with the odd number of half-waves are excited, the natural frequencies turn out to be well separated (see Figs. 4 and 5),

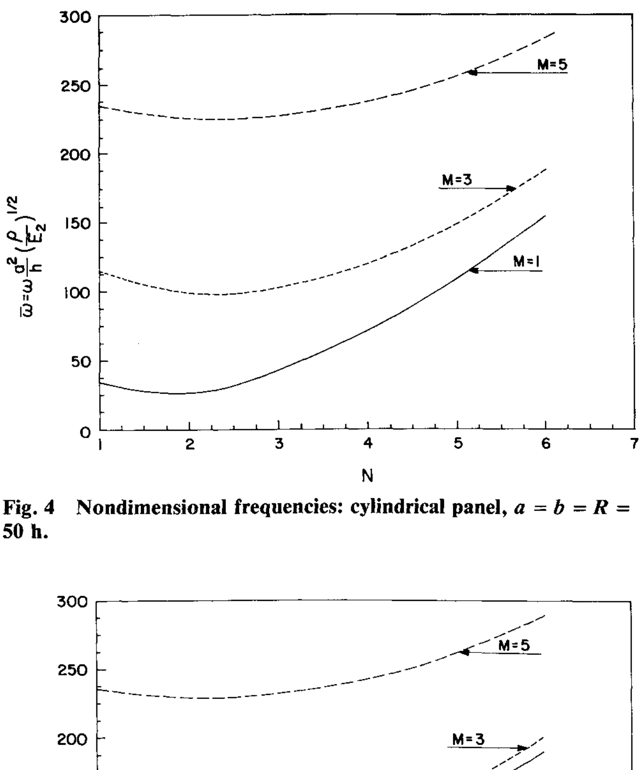

Fig. 4 Nondimensional frequencies: cylindrical panel, $a = b = R = 50$ h.

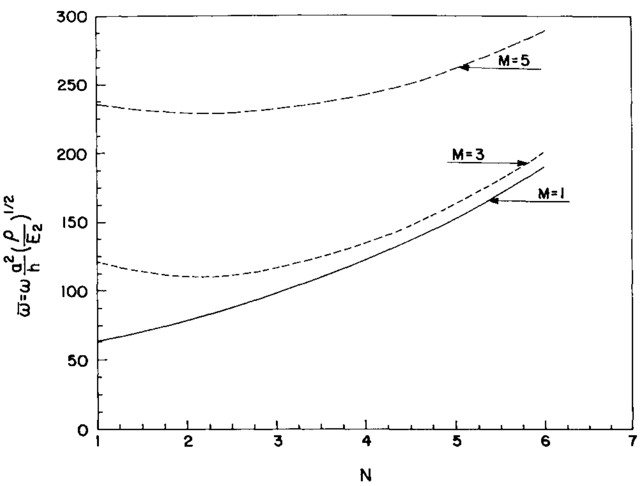

Fig. 5 Nondimensional frequencies: spherical panel, $a = b = R = 50$ h.

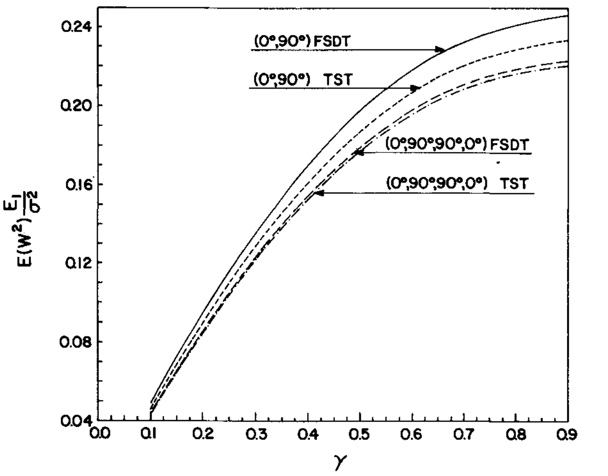

Fig. 6 Mean squares of the transverse displacements vs γ: cylindrical panel, $a = b = R = 50$ h.

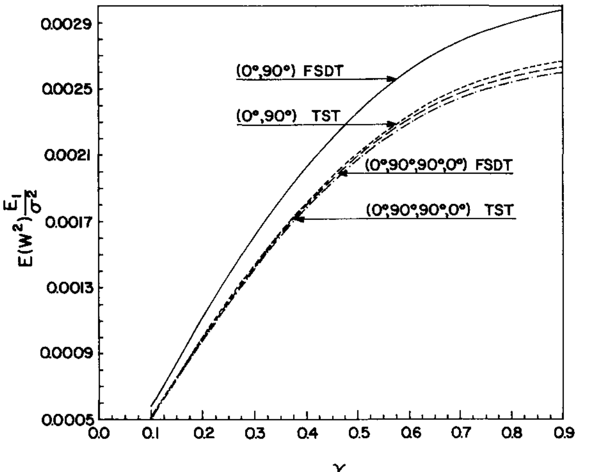

Fig. 7 Mean squares of the transverse displacements vs γ: spherical panel, $a = b = R = 50$ h.

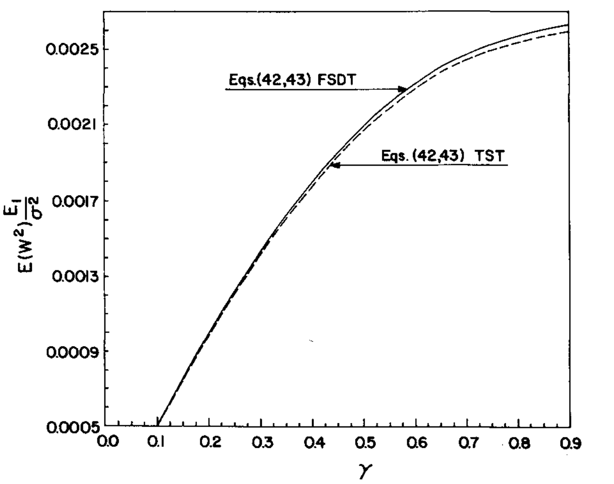

Fig. 8 Mean squares of the transverse displacements vs γ: spherical panel; comparison between the results obtained via Eq. (42) and via Eq. (43).

and, therefore, for this particular excitation the cross terms may be dropped and Eq. (39) is approximated as

$$E\left[W^2\left(\frac{a}{2}, \frac{b}{2}, \frac{a}{2}, \frac{b}{2}, 0\right)\right] \simeq \sum_{m,n} \frac{16}{m^2 n^2 J_{mn}^2} \times \int_{-\infty}^{\infty} \frac{\sigma^2}{\gamma\pi} \frac{\gamma^2}{\gamma^2 + \omega^2} \frac{\mathrm{d}\omega}{(\omega_{mn}^2 - \omega^2)^2 + 4\xi_{mn}^2 \omega_{mn}^2 \omega^2} \tag{40}$$

The solution of the integral in Eq. (40) can be solved approximately by using the Laplace asymptotic method[3]

$$\int_{-\infty}^{\infty} \frac{S_F(\omega)}{(\omega_{mn}^2 - \omega^2)^2 + 4\xi_{mn}^2 \omega_{mn}^2 \omega^2} \mathrm{d}\omega \simeq \frac{\pi S_F(\omega_{mn})}{2\xi_{mn}\omega_{mn}^3} \tag{41}$$

from which

$$\sigma_W^2 \simeq \sum_{m,n} \frac{16}{m^2 n^2 J_{mn}^2} \frac{\sigma^2}{\gamma\pi} \frac{\gamma^2}{\gamma^2 + \omega_{mn}^2} \frac{\pi}{2\xi_{mn}\omega_{mn}^3} \tag{42}$$

However, the exact solution of the integral in Eq. (40) is found to be[11]

$$\int_{-\infty}^{\infty} \frac{\sigma^2}{\pi\gamma} \frac{\gamma^2}{\gamma^2 + \omega^2} \frac{\mathrm{d}\omega}{(\omega_{mn}^2 - \omega^2)^2 + 4\xi_{mn}\omega_{mn}^2\omega^2} = \sigma^2 \left[\frac{2\xi_{mn}\omega_{mn} + \gamma}{\omega_{mn}^3 (2\xi_{mn}\omega_{mn}^2 + 4\xi_{mn}^2 \omega_{mn}^2 \gamma + 2\xi_{mn}\gamma^2)}\right] \tag{43}$$

Numerical Results

In the numerical examples, we set $\xi_{11} = 0.01$, and γ varies between $0.1\omega_{11}$ and ω_{11}. Figures 6 and 7 display the mean square of the transverse displacement of the midpoint of the cylindrical and spherical panels, respectively, obtained by using Eq. (43). Both figures display the solutions of symmetric and antisymmetric laminates within the FSDT and the classical (thin) shell theory (TST)[12]: curve 1, FSDT (0°, 90°); curve 2, TST (0°, 90°); curve 3, FSTD (0°, 90°, 90°, 0°); curve 4, TST (0°, 90°, 90°, 0°).

Figure 8 shows the comparison of two solutions of the integral in Eq. (40), for the symmetric case, within the two theories. The results indicate that the mean square calculated via the TST is lower than those calculated via the FSDT. The results obtained by the exact and the approximate solutions are very much alike.

Conclusion

The random vibrations of thick laminated cross-ply composite shells owing to the stationary random excitation are considered in this paper, apparently for the first time in the literature. For the circular shells, the cross-correlation effect turned out to be of paramount importance. For the cylindrical and spherical panels, simply supported at their edges and subjected to the loading that is uniformly distributed on the shell surface and representing an exponentially correlated stationary random process, the natural frequencies are well separated, and the cross-correlation effect can be neglected. The study of nonstationary random vibrations is under way and will be reported elsewhere.

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